

Math 60 8.3 Introduction to Functions, Day 2

Objectives 2) Find the value of a function
also called "Evaluate a Function"

* Function Notation * is the crucial concept.

↳ substitute given value

↳ use order of operations or algebra to simplify

3) Find the domain of a function from its equation

- absolute values
- polynomials
- rationals
- word problems

Function Notation

• Name of the function: a letter

Typically: $f, g, h, F, G, H, P, r, q, z$

Any letter is valid.

• Name of the independent variable

↳ recall: When writing a table, these are the values we chose first.

Typically: x, t, z

Any letter is valid so long as

name of function $\neq$ name of independent variable.

Examples

$f(x)$ say this "f of x"
↑ ↑
name of function is f the independent variable is called x

CRUCIAL: $f(x)$ does NOT mean f times x .

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Other examples of function names:

$g(x)$ "g of x"

$h(t)$ "h of t"

$F(x)$ "F of x"

$G(t)$ "G of t"

$p(x)$ "p of x"

$Q(z)$ "Q of z"

Examples

- ① Write the equation $y = \frac{3}{4}x - 1$ as a function using $y = f(x)$.

$$f(x) = \frac{3}{4}x - 1$$

replace y by $f(x)$

- ② Write the equation $y = 2t^2 - 3$ as a function using $y = g(t)$

$$g(t) = 2t^2 - 3$$

replace y by $g(t)$

- ③ Write the equation $y = 2t^2 - 3$ as a function using $y = P(z)$.

$$P(z) = 2z^2 - 3$$

replace y by $P(z)$
replace t by z

Find the value of the function. (Evaluate the function.)

④ $f(x) = -2x^2 + 5x - 4$

find $f(-3)$

← say this "f of -3"

step 1: Replace x by -3 , using parentheses

$$\begin{array}{ccccccc} f(-3) & = & -2(-3)^2 & + & 5(-3) & - & 4 \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{replace} & & \text{replace} & & \text{replace} & & \\ x & & x & & x & & \end{array}$$

step 2: Evaluate using the order of operations.

$$\begin{aligned} f(-3) &= -2(9) - 15 - 4 \\ &= -18 - 15 - 4 \\ &= \boxed{-37} \end{aligned}$$

What have we found?

$$f(-3) = -37$$

means that when the x -coordinate is -3 , the y -coordinate (on the graph of $y = -2x^2 + 5x - 4$) is -37 .

This means that the ordered pair $(-3, -37)$ is a point on the graph of $f(x) = -2x^2 + 5x - 4$.

⑤ $h(t) = \frac{3}{2}t - 4$

find $h(2)$

$$\begin{aligned} h(2) &= \frac{3}{2}(2) - 4 \\ &= 3 - 4 \\ &= \boxed{-1} \end{aligned}$$

$h(2) = -1$ means $(2, -1)$ is a point on the graph.

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⑥ $g(x) = \frac{3}{2}x - 4$
find $g(12)$

$$\begin{aligned} g(12) &= \frac{3}{2}(12) - 4 \\ &= 18 - 4 \\ &= \boxed{14} \end{aligned}$$

Notice: This is the same function as #5, just using different letters

$g(12) = 14$ means $(12, 14)$ is a point on the graph.

Write each statement (given in function notation) as an ordered pair on the graph.

⑦ $f(-1) = -2$
↑
x-value is -1 y-value is -2

$$\boxed{(-1, -2)}$$

⑧ $g(47) = -6$

$$\boxed{(47, -6)}$$

Write each ordered pair using $y = f(x)$ notation.

⑨ $(3, 8)$

$$\boxed{f(3) = 8}$$

x-value inside parentheses
y-value on RHS.

⑩ $(-9, 0)$

$$\boxed{f(-9) = 0}$$

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Write each ordered pair using $y = g(x)$ notation.

⑪ $(1, -6)$
↑ x-coord ↗ y-coordinate

$y = g(1)$ replace x by 1

$-6 = g(1)$ replace y by -6 .

$\boxed{g(1) = -6}$ switch order

⑫ $(-2, -5)$

$\boxed{g(-2) = -5}$

○ Write each spoken name using function notation.

⑬ "f of x"

$\boxed{f(x)}$

⑭ "g of t"

$\boxed{g(t)}$

⑮ "R of z"

$\boxed{R(z)}$

○

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Find the value of the function.

(16) $f(x) = 3 - 2x$

Find $f(z+3)$.

step 1: Replace x by $(z+3)$, using parentheses.

$$f(z+3) = 3 - 2(z+3)$$

step 2: simplify

$$f(z+3) = 3 - 2z - 6$$

$$f(z+3) = \boxed{-2z - 3}$$

(17) $f(x) = 3 - 2x$

Find $f(z) + f(3)$.

step 1: replace x by z replace x by 3

* use the function twice *

$$f(z) + f(3) = 3 - 2z + 3 - 2(3)$$

step 2: simplify

$$= 3 - 2z + 3 - 6$$

$$= \boxed{-2z}$$

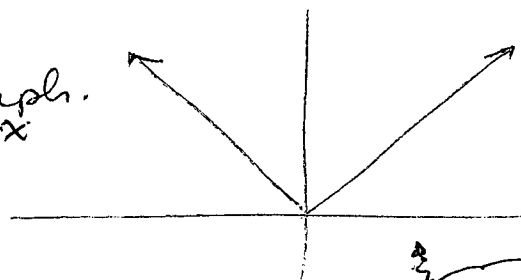
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Find the domain of the function. a) set notation
b) interval notation

⑧ $f(x) = |x|$

step 1: Graph or imagine the graph.
OR: imagine substituting values of x

The domain (x -values)
is all real numbers.



any
value of
 x has a
 y -coord

step 2: Write using notation requested.

set notation $\{x \mid x \text{ is a real } \# \}$

interval notation $(-\infty, \infty)$

⑨ $g(x) = 573 + 46|x - 24|$

at its core, this function is
just a V-shaped absolute value
function.

set notation $\{x \mid x \text{ is a real } \# \}$

interval notation $(-\infty, \infty)$

In Summary: The domain of any function containing
an absolute value (in Math 60) is all real $\#$ s.

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Find the domain of the function a) set notation
b) interval notation.

$$\textcircled{10} \quad h(t) = 3t^4 - 7t^3 + 5t^2 - t - 31$$

↑ ↑ ↑ ↑

notice: all positive exponents.
any value of t (our independent variable)
can be evaluated.

domain $\{t : t \text{ is a real \#}\}$
 $(-\infty, \infty)$

Definition: An expression (like $3t^4 - 7t^3 + 5t^2 - t - 31$) which is the sum or difference of the variable (with coefficients) having only positive exponents is called a polynomial.

$$\textcircled{11} \quad f(x) = 3x^{100} + 2x^2 + 1$$

↑ ↑

any value of x can be evaluated

domain $\{x : x \text{ is a real \#}\}$
 $(-\infty, \infty)$

In summary: The domain of any function given by a polynomial is all real \#s.

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Find the domain of the function

a) set notation

b) interval notation.

(12) $f(x) = x^{-1}$

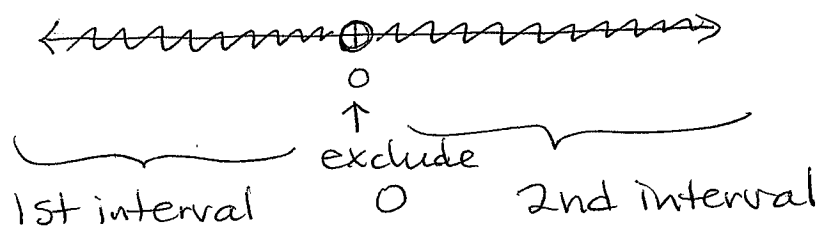
Remember: $x^{-1} = \frac{1}{x}$.

If $x=0$ we can have an undefined result $\rightarrow$ not on the graph!

$$f(0) = \frac{1}{0} = \text{undefined.}$$

But all other values of x can be divided.

Domain $\boxed{\{x : x \text{ is a real \#}, x \neq 0\}}$ set notation



$\boxed{(-\infty, 0) \cup (0, \infty)}$ interval notation.

Definition: A function which is a fraction with a variable expression in the denominator is called a rational function.

(13) $h(x) = \frac{x-1}{x^2-4}$

Step 1: Find the values of the variable that make the expression undefined.

{This is a review question from 7.1 except we are now using function notation.}

Undefined means divide by zero

$\Rightarrow$ part by which we divide is denominator.

Set the denominator equal to zero.

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$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x-2=0 \quad x+2=0$$

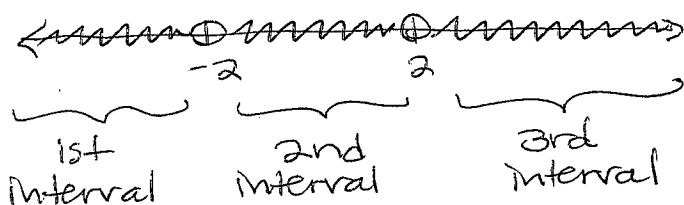
$$x=2 \quad x=-2$$

← exp 2 means this is a quadratic equation.
Solve by factoring.
set factors = 0.
isolate x

step 2: The values we found in step 2 do not work.
We exclude them from the domain.

set notation $\{x \mid x \text{ is a real \#, } x \neq 2, x \neq -2\}$

To find interval notation, graph the domain:



$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

* Be sure to use parentheses $()$ to exclude endpoints.

(14) $g(x) = \frac{2x-1}{2x^2-x-1}$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(2x+1)(x-1) = 0$$

$$2x+1=0$$

$$x-1=0$$

$$2x=-1$$

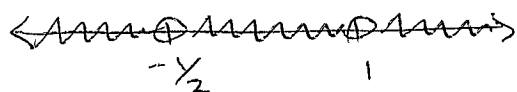
$$x=1$$

$$x = -\frac{1}{2}$$

set denom = 0.

$$\begin{array}{r} -2 \\ -2 \times 1 \\ -1 \end{array}$$

$\{x \mid x \text{ is a real \#, } x \neq -\frac{1}{2}, x \neq 1\}$



$$(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 1) \cup (1, \infty)$$

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Find the domain of the function.

- (15) The speed v of a car as a function of the time t (in seconds) for the first minute after acceleration is $v(t) = 5t$.

The independent variable (or input) time only makes sense if t is zero or positive.

$$t \geq 0$$

This function is valid only for the first minute = 60 seconds, so

$$t \leq 60.$$

Domain is $\{t | 0 \leq t \leq 60\}$ or $[0, 60]$

- (16) In 2010, the Deepwater Horizon oil explosion spilled millions of gallons of oil into the Gulf of Mexico. The oil slick on the surface is the shape of a circle, with area $A(t) = 0.25\pi t^2$ where A is in square miles and t is the number of days since the explosion. Evaluate $A(30)$ and explain what it means. Round to the nearest hundredth.

$$\begin{aligned} A(30) &= 0.25\pi (30)^2 \\ &= 706.858 \end{aligned}$$

706.86 square miles after 30 days of oil

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Objectives

- 1) Understand and use function notation correctly
- 2) Evaluate a function or find the value of a function
- 3) Find the domain of a function
 - a. Absolute value functions
 - b. Polynomial functions
 - c. Rational functions
 - d. Context given in a word problem

$y = f(x)$ is pronounced "y equals f of x"

****THIS DOES NOT MEAN f TIMES x ****

Examples and Practice

- 1) Write the equation $y = \frac{3}{4}x - 1$ as a function using $y = f(x)$
- 2) Write the equation $y = 2t^2 - 3$ as a function using $y = g(t)$
- 3) Write the equation $y = 2t^2 - 3$ as a function using $y = P(z)$

Find the value of the function. (same as: Evaluate the function.)

- 4) Given $f(x) = -2x^2 + 5x - 4$, find $f(-3)$

- 5) Given $h(t) = \frac{3}{2}t - 4$, find $h(2)$

- 6) Given $g(x) = \frac{3}{2}x - 4$, find $g(12)$

Write each statement (given in function notation) as an ordered pair on the graph.

7) $f(-1) = -2$

8) $g(47) = -6$

Write each ordered pair using $y = f(x)$ notation.

9) $(3, 8)$

10) $(-9, 0)$

Write each ordered pair using $y = g(x)$ notation.

11) $(1, -6)$

12) $(-2, -5)$

Write each spoken name using function notation.

13) "*f of x*"

14) "*q of t*"

15) "*R of z*"

Find the value of the function. (same as: Evaluate the function.)

16) $f(x) = 3 - 2x$, find $f(z + 3)$

17) $f(x) = 3 - 2x$, find $f(z) + f(3)$

Find the domain of the function. Write the answer using

a. Set notation

b. Interval notation

18) $f(x) = |x|$

19) $g(x) = 573 + 46|x - 24|$

20) $h(t) = 3t^4 - 7t^3 + 5t^2 - t - 31$

21) $f(x) = 3x^{100} + 2x^2 + 1$

22) $f(x) = x^{-1}$

23) $h(x) = \frac{x-1}{x^2-4}$

24) $g(x) = \frac{2x-1}{2x^2-x-1}$

25) The speed v of a car as a function of time t (in seconds) for the first minute after acceleration is $v(t) = 5t$.

Solve the problem.

26) In 2010, the Deepwater Horizon oil explosion spilled millions of gallons of oil into the Gulf of Mexico. The oil slick on the surface is the shape of a circle, with area $A(t) = 0.25\pi t^2$ where A is the area in square miles and t is the number of days since the explosion. Evaluate $A(30)$ and explain what it means. Round to the nearest hundredth.